Case of a forced spring-mass-system with damping
On pages 193, 194 of your book you have seen you have seen the solution of the following initial value problem: $u'' + .125u' + u = 0,\ \ u(0) = 2,\ u'(0) = 0$. As we noted this equation models the motion of a spring mass system with damping and with no external force.
The solution was tedious, but we did it in class to understand the workings of the solutions like it. Using Maple, the solution could have been easily worked out via the following command:

```
> dsolve({diff(u(t),t$2)+.125*diff(u(t),t)+u(t)=0,u(0)=2,
D(u)(0)=0},u(t));
```

$$u(t) = \frac{2}{255} e^{(-1/16\, t)} \sin\left(\frac{1}{16}\sqrt{255}\, t\right)\sqrt{255} + 2\, e^{(-1/16\, t)} \cos\left(\frac{1}{16}\sqrt{255}\, t\right)$$

Now we are up against the following situation: $F(t) \neq 0$
That is, there is an external force $F(t) = 3\cos(2t)$ applied to the system. The situation then can be modeled by the following differential equation:

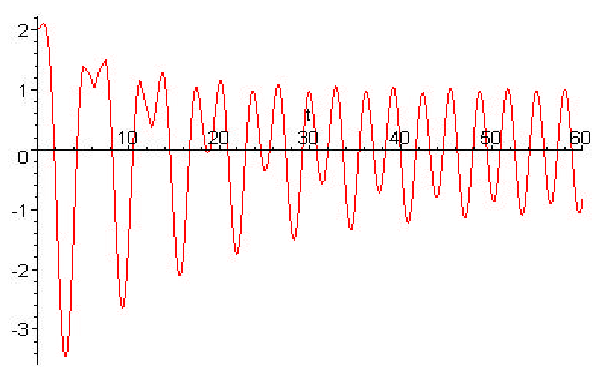
$u'' + .125u' + u = 3\cos(2t),\ \ u(0) = 2,\ u'(0) = 0$. This equation too can be solved by hand but the calculations involved are quite tedious and besides we need to see the plot to see what the external force does to the solution, because we know that the solution to the corresponding homogeneous tends to zero as *t* tend to infinity. For this we use the following commands: First to solve the initial value problem we have:

```
>
dsolve({diff(u(t),t$2)+.125*diff(u(t),t)+u(t)=3*cos(2*t),u(
0)=2, D(u)(0)=0},u(t));
```

$$u(t) = \frac{2}{1479} e^{(-1/16\, t)} \sin\left(\frac{1}{16}\sqrt{255}\, t\right)\sqrt{255} + \frac{434}{145} e^{(-1/16\, t)} \cos\left(\frac{1}{16}\sqrt{255}\, t\right) + \frac{12}{145}\sin(2\, t) - \frac{144}{145}\cos(2\, t)$$

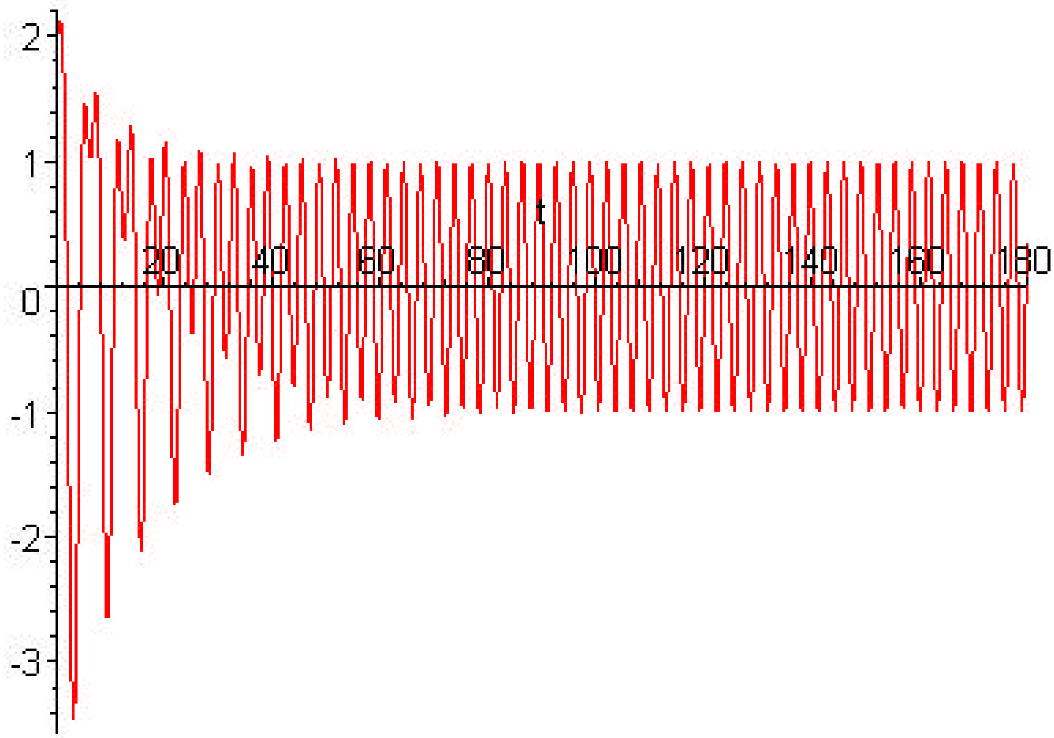
Using the following command we plot the graph of the solution:

```
> plot(exp(-
t/16)*(2/1479)*sin(sqrt(255)*t/16)*sqrt(255)+434/145*exp(-
t/16)*cos(sqrt(255)*t/16)+12/145*sin(2*t)-
144/145*cos(2*t),t=0..60);
```



We note that the amplitude of the vibrations becomes smaller but does not disappear as in the case of the unforced system. Just to be certain let us increase the range of t :

```
> plot(exp(-
t/16)*(2/1479)*sin(sqrt(255)*t/16)*sqrt(255)+434/145*exp(-
t/16)*cos(sqrt(255)*t/16)+12/145*sin(2*t)-
144/145*cos(2*t),t=0..180);
```



This picture seems to tell us that while the transient part was active there were somewhat wild fluctuations but as time passes the effect of the u_c part of the solution tend to zero and thanks to the external force a steady vibration sets in. The part of the graph of the solution with a fixed amplitude is the graph of the steady state solution.

In our case this happens to be: $u_S = \frac{12}{145}\sin(2t) - 144\cos(2t)$

```
> plot(12/145*sin(2*t)-144/145*cos(2*t),t=80..180);
```