QUESTION (HD0403): If R is a Prufer v-multiplication domain such that every maximal t-ideal of R is a maximal ideal, must R be a Prufer domain?

ANSWER: If R is a Prufer v-multiplication domain (PVMD) such that every maximal t-ideal of R is a maximal ideal then R is **not necessarily** a Prufer domain. Heinzer and Lantz in [Integral domains that lose ideals in overrings. Pacific J. Math. 145 (1990), no. 2, 223–238 MR 91k:13012] construct an example (Example 3.2) of a non-Bezout GCD domain in which every maximal t-ideal is a maximal ideal. Indeed as a GCD domain is a PVMD and a Prufer GCD domain is Bezout, this is a good example to answer your question. If however, R is a PVMD and every maximal ideal of R is a t-ideal then R is a Prufer domain. This is easy to establish since it is well known that a domain R is a Prufer domain if and only if R_M is a valuation domain for every maximal ideal M of R and R is a PVMD if and only if R_P is a valuation domain for every maximal t-ideal P of R.

(This question is related to Proposition 4.4 part 3(c) of [Mott and Zafrullah, On Prufer v-multiplication domains, Manuscripta Math. 35 (1981), no. 1-2, 1–26] which is erroneous. I have already pointed out this error in print elsewhere.) Also, on this web page click on Multiplicative Ideal Theory. At the end of that section you will find copies of my old papers. The above mentioned Mott-Zafrullah paper is one of them. Check to see that I have crossed out that part of Proposition 4.4.)

(Comments dated November 10, 2004). This question is a somewhat modified form of a question asked by Mohammad Sakhdari of the University of Kashan, Kashan, Iran).

(Evan Houston helped straighten the presentation of this answer. Zafrullah)