

QUESTION: (HD0804) Call an irreducible element an atom. Some authors define an atomic domain as an integral domain in which every nonzero nonunit is a product of atoms and some define an atomic domain as one in which every nonzero nonunit is expressible as a product of atoms. Is there a difference?

ANSWER: If a nonzero nonunit is expressible as a product of atoms, it is a finite product of atoms once expressed. On the other hand when we claim that a nonzero nonunit is a product of atoms we have to prove that it is expressible as a product of atoms to establish that it is a product of atoms. So, the two statements are equivalent.

I asked my usual experts. One of them said that, as I pointed out, the two statements are the same. Another told me that it is a matter of semantics, otherwise the two statements are the same. The third one said that it was a "stupid question to ask". Do not listen to the third expert. No question is stupid or frivolous. A question usually indicates interest and interest cannot be bad.

I would however add the following to my answer. In either definition it is tacitly assumed that products are finite. Well it is an underlying assumption in Algebra that we can only perform finitely many operations. However algebra often gets its material and problems from disciplines that can find their way around "finite" by taking limits or by involving sets. For example it is well known that in the ring E of entire functions every nonzero nonunits is expressible as associates of a countable product of powers of prime elements (see the two Theorems at the bottom of page 146 of Gilmer [Multiplicative Ideal Theory, Dekker, 1972]) and note that "countable" includes "countably infinite".

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