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ON T-INVERTIBILITY II

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Introduction: Several recent papers ([5], [11], [14], [16], and [17]) have characterized Krull domains in terms of the t-invertibility of some or all of the fractional ideals. In this paper we undertake a more general study of t-invertibility, presenting instances where it occurs naturally and where it has interesting applications.

Throughout this work D will denote an integral domain with quotient field K. We assume familiarity with properties of star operations as found in [6, Sections 32 and 34] or [12]. Of particular interest are the v- and t-operations. Recall that the v-operation on D is defined as follows: for a fractional ideal I of D, $I_v = (I^{-1})^{-1}$. Then I_t is defined to be $\cup\{J_v$: J is a nonzero finitely generated subideal of I}.

In the first section we prove that t-invertible prime t-ideals are always maximal t-ideals. We prove in Theorem 1.4, moreover, that if P is an upper to zero, that is, if P is a nonzero prime of D[x] which is contracted from K[x], then P is t-invertible $\Leftrightarrow$ P is a maximal t-ideal $\Leftrightarrow$ $c(P)_t = D$. (Here, c(P) denotes the ideal of D generated by the coefficients of all the polynomials in P.)

A good deal of the remainder of the paper is devoted to consequences of these results. For example, in section 2 we give a simple proof of the fact, due to Glaz and Vasconcelos, that the property of being an H-domain is preserved upon passage from D to D[x]. We also prove that D is a Krull domain ⇔ every maximal t-ideal of D is t-invertible and has height one. In a more general vein we show that D is integrally closed ⇔ every t-invertible t-ideal of D[x] can be written in the form f I[x] for some polynomial $f \in D[x]$ and some ideal I of D. A simple consequence of this is that D is integrally closed ⇔ $\mathcal{C}(D) \cong \mathcal{C}(D[x])$. Here $\mathcal{C}(D)$ denotes the t-class group of D, that is, $\mathcal{C}(D) = \mathcal{T}(D)/\mathcal{P}(D)$, where $\mathcal{T}(D)$ (respectively, $\mathcal{P}(D)$) denotes the group of t-invertible t-ideals (respectively, the group of nonzero principal ideals) of D.

Section 3 is devoted to the study of domains D for which every upper to zero in D[x] is a maximal t-ideal; we dub these rings UMT-domains. We show that D is a UMT-domain ⇔ every prime ideal of $D[x]_N$ is extended from D, where $N = \{f \in D[x]: c(f)_v = D\}$. We also characterize Noetherian UMT-domains; they are precisely the Noetherian domains in which every prime t-ideal has height one.

Finally, in section 4 we generalize part of Theorem 1.4 to the following statement: if B is an ideal of D[x] for which c(B) is t-invertible, then B is t-invertible.

Section 1. Basic results.

PROPOSITION 1.1. If M is a maximal t-ideal of D[x] with $M \cap D \neq 0$, then $M = (M \cap D)[x]$.

Proof: It suffices to show that $c(M) \subseteq M$, for then $M \subseteq c(M)[x] \subseteq (M \cap D)[x]$. By [9, Proposition 4.3] $c(M)_t[x] = (c(M)[x])_t$. Hence if $c(M) \not\subseteq M$ then, since M is a maximal t-ideal, $c(M)_t[x] = (c(M)[x])_t = D[x]$. It follows that $c(M)_t = D$, whence there is an element $g \in M$ with $c(g)_v = D$. (This is a well-known argument. First choose $g_1,\dots,g_k \in M$ with $c(g_1,\dots,g_k)_v = D[x]$. Then for appropriately chosen exponents, the polynomial $g = g_1 + x^{r_2}g_2 + \cdots x^{r_k}g_k$ satisfies $c(g)_v = D$.) But then $(a,g)_v = D[x]$ for any nonzero element a of $M \cap D$ [9, Lemma 4.4], contradicting that M is a proper t-ideal.

LEMMA 1.2. Let P be a nonzero prime ideal of D. If $PP^{-1} \not\subseteq P$, then P is a minimal prime of a principal ideal (a) of D, and P = (a):b for some $b \in D$.

Proof: Since $PP^{-1} \not\subseteq P$ there are elements $a \in P$ and $u \in P^{-1}$ with $au \notin P$. Clearly, $P \subseteq (a):au$. If $x \in D$ with $xau \in (a)$, then $x \in P$, since $a \in P$ and $au \notin P$. Hence $P = (a):au$. Since for any prime $Q \subseteq P$ with $a \in Q$, we have $(a):au \subseteq Q$, P is minimal over a.

We now state our first result on the consequences of t-invertibility. Part 3) of Proposition 1.3 has also been obtained by S. Gabelli [5].

PROPOSITION 1.3. If P is a t-invertible prime t-ideal of D, then

1) P is a minimal prime of a principal ideal,

2) P has the form (a):b for some $a,b \in D$, and

3) P is a maximal t-ideal.

Proof: Since P is a t-invertible t-ideal, $PP^{-1} \not\subseteq P$, and conclusions 1) and 2) follow from Lemma 1.2. To see that P is a maximal t-ideal, let M be an ideal of D which contains P, and pick $c \in M - P$. Since P is a t-invertible prime t-ideal, $P = I_v$ for some finitely generated ideal I of D. Now suppose $u \in (I,c)^{-1}$. Then $ucI \subseteq I \subseteq P \Rightarrow uI \subseteq P \Rightarrow uP = uI_v \subseteq P$. Since P is t-invertible, this implies that $u \in D$. Hence $(I,c)^{-1} = D$, and, since $(I,c) \subseteq M$, M is not a t-ideal. Therefore, P is a maximal t-ideal, as claimed.

It is easy to produce examples of maximal t-ideals which are not t-invertible, but the two conditions are equivalent for a certain class of prime ideals, as we now show.

THEOREM 1.4. Let P be an upper to 0 in D[x]. The following statements are equivalent.

1) P is a maximal t-ideal.

2) P is t-invertible.

3) $c(P)_t = D$. (In this case it is easy to produce $g \in P$ such that $c(g)_v = D$.)

Proof: 3) ⇒ 1). Suppose that P is not a maximal t-ideal, and let M be a maximal t-ideal of D[x] which contains P. Since the containment is proper, we have that $M \cap D \neq 0$. By Proposition 1.1, $M = (M \cap D)[x]$. Since $P \subseteq M$, c(P) is contained in the t-ideal $M \cap D$, so that $c(P)_t \neq D$.

1) ⇒ 3). Since $P \subsetneq c(P)[x]$, we have $c(P)_t = D$.

2) ⇒ 1). This follows from Proposition 1.3.

1) ⇒ 2). Since P is a maximal t-ideal, either $PP^{-1} \subseteq P$ or P is t-invertible. By 1) ⇒ 3) there is an element $g \in P$ with $c(g)_v = D$. By [8, Proposition 1.8] there is a nonzero element a of D with $aP \subseteq (f)$, where $P = fK[x] \cap D[x]$. Clearly, $\frac{a}{f} \in P^{-1}$, but $\frac{a}{f}P \nsubseteq P$. Hence $PP^{-1} \nsubseteq P$, and the proof is complete.

COROLLARY 1.5. If P is an upper to 0 which is also a maximal t-ideal, then P is divisorial. In fact, $P = (f,g)_v$, where $f,g \in P$ are such that $P = fK[x] \cap D[x]$ and $c(g)_v = D$.

Proof: Clearly, $(f,g)_v \subseteq P$. Let $h \in P$. Then $ah \in (f)$ for some nonzero $a \in D$. Thus $h(a,g) \subseteq (f,g)$. Applying the v-operation to both sides and using the fact that $(a,g)_v = D[x]$, we have $h \in (f,g)_v$, and the proof is complete.

Remark. Uppers to zero which are not divisorial exist [8, Example 2.3].

Section 2. Applications.

In [7] Glaz and Vasconcelos define an H-domain to be a domain D in which every ideal I with $I^{-1} = D$ has the property that $J^{-1} = D$ for some finitely generated subideal J of I. They then show that the H-domain property is preserved upon passage to the polynomial ring. We begin this section with a simple proof of this fact.

PROPOSITION 2.1 . If D is an H-domain, then so is D[x].

Proof: By [11, Proposition 2.4] it suffices to show that every maximal t-ideal of D[x] is divisorial. Accordingly, let P be a maximal t-ideal. If $P \cap D \neq 0$ then $P = (P \cap D)[x]$ by Proposition 1.1. In this case $P \cap D$ is a maximal t-ideal of D. Since D is an H-domain, $P \cap D$ is divisorial, and it follows easily [9, Proposition 4.3] that P is divisorial. If $P \cap D = 0$ then P is divisorial by Corollary 1.5.

PROPOSITION 2.2. Let P be an upper to 0 in D[x] which is a maximal ideal. Then P is invertible. (This sharpens [15, Exercise 20, p. 75].)

Proof: Since P is t-invertible $PP^{-1} \nsubseteq P$. Since P is a maximal ideal, this implies that $PP^{-1} = D[x]$, and P is invertible.

We next discuss t-invertibility in Prüfer v-multiplication domains (PVMD's). Recall that a PVMD is an integral domain in which each nonzero finitely generated ideal is t-invertible. (The usual definition is a domain in which the v-ideals of finite type form a group under v-multiplication, but it is well known that the two definitions are equivalent.)

PROPOSITION 2.3. Let D be a PVMD, and let P be a prime ideal of D.

Then P is a t-invertible prime t-ideal $\Leftrightarrow$ P is of the form (a):b.

Proof: By [21, Lemma 8] if P = (a):b then P is a v-ideal of finite type; that is, $P = J_v$ for some finitely generated subideal J of P. Since D is a PVMD, J, and hence P also, is t-invertible. The converse follows from Proposition 1.3.

We now apply the preceding results to obtain characterizations of Krull domains. Some of Proposition 2.4 is contained in [14, Theorem 3.6], but we feel that our approach is sufficiently different to merit inclusion here.

PROPOSITION 2.4. The following statements are equivalent.

1) D is a Krull domain.

2) D is a PVMD in which every minimal prime of a principal ideal has the form (a):b.

3) Every minimal prime of a nonzero principal ideal is t-invertible.

4) Every maximal t-ideal is t-invertible and has height one.

Proof: 1) ⇒ 2) This is well known.

2) ⇒ 3) This follows from Proposition 2.3.

3) ⇒ 4) Let P be a maximal t-ideal. Then P contains a prime Q which is minimal over a nonzero principal ideal. By 3) Q is t-invertible, whence by Proposition 1.3 Q is a maximal t-ideal. Thus P = Q and P is t-invertible. A similar argument shows that ht(P) = 1.

4) ⇒ 1) Since every t-ideal is contained in a maximal t-ideal, it follows that every prime t-ideal of D is t-invertible. By [11, Theorem 2.3] D is a Krull domain.

Definition. A fractional ideal B of D[x] is said to be extended from D if $B = fI[x]$ for some $f \in K[x]$ and some fractional ideal I of D. Note that if $B \cap D \neq 0$, then f is a constant and B is extended in the usual sense.

In [3, Theorem 1] Brewer and Costa showed that D is seminormal ⇔ each invertible fractional ideal of D[x] is extended from D, and Querré [18] proved that D is integrally closed ⇔ each divisorial ideal of D[x] is extended from D. (See [20] for a particularly simple treatment of this result.) We show below that D is integrally closed ⇔ every t-invertible t-ideal of D[x] is extended from D. First we record a lemma whose proof is almost trivial and hence omitted.

LEMMA 2.5. Let $P = fK[x] \cap D[x]$ be an upper to 0. Then P is extended from D $\Leftrightarrow$ $P = fc(f)^{-1}D[x]$.

PROPOSITION 2.6. The following statements are equivalent.

1) D is integrally closed.

2) Every divisorial ideal of D[x] is extended from D.

3) Every t-invertible t-ideal of D[x] is extended from (a necessarily t-invertible t-ideal of) D.

4) Every t-invertible upper to 0 in D[x] is extended from D.

Proof: 1) $\Rightarrow$ 2) This follows from [18, Lemme 3.2].

2) $\Rightarrow$ 3) Let B be a t-invertible t-ideal of D[x]. Then B is divisorial, so by 2) $B = gA[x]$ for some element $g \in D[x]$ and some fractional ideal A of D. Since $A[x] = \frac{1}{g}B$ is a t-invertible t-ideal, so is A.

3) $\Rightarrow$ 4) Trivial.

4) $\Rightarrow$ 1) Let $u \in K$ be integral over D, and let $P = fK[x] \cap D[x]$, where $f(x) = x - u$. Since u is integral over D, there is a monic polynomial $g \in P$. Write $g = fk$ with $k \in K[x]$. By Theorem 1.4 P is t-invertible, whence $P = fc(f)^{-1}D[x]$ by hypothesis and Lemma 2.5. Hence $k \in c(f)^{-1}D[x] \Rightarrow ku \in D[x]$. Since k is monic this puts $u \in D$. Therefore, D is integrally closed.

In [2] the authors define the class group $\mathcal{C}(R)$ of an integral domain R to be the quotient group $\mathcal{T}(R)/\mathcal{P}(R)$, where $\mathcal{T}(R)$ is the group of t-invertible t-ideals of R and $\mathcal{P}(R)$ is the group of principal ideals of R. As an easy consequence of Proposition 2.6, we record the following result of Gabelli [4].

COROLLARY 2.7. D is integrally closed $\Leftrightarrow \mathcal{C}(D) \cong \mathcal{C}(D[x])$.

Proof: The map $\psi:\mathcal{C}(D) \to \mathcal{C}(D[x])$ given by $\psi(I\mathcal{P}(D)) = I[x]\mathcal{P}(D[x])$ is easily seen to be a monomorphism for any domain D [4]. By 1) $\Leftrightarrow$ 3) of Proposition 2.6 , ψ is an epimorphism precisely when D is integrally closed.

Section 3. UMT-domains.

In this section we study the class of domains all of whose uppers to zero satisfy the equivalent conditions of Theorem 1.4. These domains are related to several conjectures that have appeared in the literature.

Definition. A UMT-domain is an integral domain D in which every upper to zero in D[x] is a maximal t-ideal (hence t-invertible).

Examples of UMT-domains include PVMD's (cf. Proposition 3.2 below) and one dimensional Noetherian domains (Theorem 3.7). Other examples can be constructed as follows. Let D be a UMT-domain, and let R be an overring with the property that, whenever A is a finitely generated ideal of D with $A^{-1} = D$, then $R:AR = R$. Then R is also a UMT-domain. To see this let Q be an upper to zero in R[x]. Then $Q \cap D[x]$ is an upper to zero in D[x], whence, since D is a UMT-domain, $Q \cap D[x]$ contains f with $c(f)_v = D$ (Theorem 1.4). By the hypotheses on R, we have that $c_R(f)_v = R$, so R is also a UMT-domain. In particular, any localization of a UMT-domain is again a UMT-domain (cf. [21, Lemma 4]). However, it is not the case that a domain D which is locally UMT, in the sense that D_M is a UMT-domain for each maximal ideal M of D, is necessarily a UMT-domain. In view of Proposition 3.2 below, this may be seen by considering any locally PVMD which is not a PVMD. Some explicit examples are given in [22].

Notation. For a domain D, let $N = \{f \in D[x]: c(f)_v = D\}$.

THEOREM 3.1. D is a UMT-domain ⇔ every prime of $D[x]_N$ is extended from D (in the usual sense).

Proof: Suppose that D is a UMT-domain, and let $Q' = QD[x]_N$ be prime in $D[x]_N$, where Q is prime in D[x]. Set $P = Q \cap D$. If $Q \neq P[x]$, pick $f \in Q - P[x]$. By [19, Theorem A] there is an upper to zero Q′ with $f \in Q' \subseteq Q$. By hypothesis $\exists\, g \in Q'$ with $c(g)_v = D$. However, this contradicts the fact that $Q \cap N = \emptyset$. Therefore, $Q = P[x]$, and Q′ is extended from D.

Conversely, suppose that D is not a UMT-domain; then there is an upper P to 0 with $P \cap N = \emptyset$. It is easy to see that $PD[x]_N$ is a prime in $D[x]_N$ which is not extended from D.

Using [10, Proposition 2.6] we can state the following result.

PROPOSITION 3.2. D is a PVMD ⇔ D is an integrally closed UMT-domain.

Remark. Kang [13, Theorem 3.1] has shown that D is a PVMD ⇔ every ideal of $D[x]_N$ is extended from D.

Definition. A domain D is said to be quasi-coherent (cf. [1]) if I^{-1} is finitely generated for each nonzero finitely generated ideal I of D.

PROPOSITION 3.3. If D is a quasi-coherent UMT-domain, then the integral closure D′ of D is a PVMD.

Proof: We first show that whenever I is a finitely generated ideal of D for which $I^{-1} = D$, then $D':(ID') = D'$. Let I be as stated. Suppose that $u \in K$ satisfies $uID' \subseteq D'$. Since I is finitely generated, $J = D[uI]$ is a finitely generated fractional ideal of D such that $uIJ \subseteq J$.

Applying the v-operation to both sides we have $uJ_v \subseteq J_v$. Since D is quasi-coherent, J_v is finitely generated, whence $u \in D'$. Thus $D':(ID') = D'$, as claimed. Now let P be an upper to zero in D'[x]. Then $P \cap D[x]$ is an upper to zero in D[x], and, since D is a UMT-domain, $P \cap D[x]$ contains an element f with $c_D(f)_v = D$. By what was shown above $D':c_{D'}(f) = D'$. Therefore, D' is an integrally closed UMT-domain, which implies that D' is a PVMD.

REMARK 3.4. In [7] Glaz and Vasconcelos conjecture that the integral closure of a one dimensional coherent domain is a Prüfer domain. Since one dimensional PVMD's are Prüfer domains, Proposition 3.3 seems to provide evidence for the conjecture. This is somewhat illusory, however, since, as is shown in Corollary 3.6 below, any one dimensional domain with Prüfer integral closure is already a UMT-domain. At any rate it is conceivable that the following more general statement is true: the integral closure of any quasi-coherent domain is a PVMD. That is, it is possible that the UMT hypothesis is unnecessary in Proposition 3.3.

It is also possible that the hypothesis of quasi-coherence is superfluous. As evidence we cite the fact, proved in Corollary 3.6, that the integral closure of any one dimensional UMT-domain is a Prüfer domain.

THEOREM 3.5. If D is a UMT-domain in which each maximal ideal is a t-ideal, then the integral closure D' of D is a Prüfer domain. Conversely, if D' is a Prüfer domain, then D is a UMT-domain.

Proof: Assume that D is a UMT-domain in which each maximal ideal is a t-ideal. Let M' be a maximal ideal of D'. We must show that $D'_{M'}$ is a valuation domain. For this it suffices by [6, Theorem 19.15] to show that if P' is an upper to zero in D'[x], then $P' \nsubseteq M'[x]$. If $P' \subseteq M'[x]$, then $P \subseteq M[x]$, where $P = P' \cap D[x]$ and $M = M' \cap D$. However, this is impossible since by hypothesis P is a maximal t-ideal of D[x] and M[x] is a t-ideal of D[x] (since M is a t-ideal of D). Therefore, D' is a Prüfer domain.

Now suppose that D′ is a Prüfer domain. Let P be an upper to zero in D[x]. We claim that c(P) = D. If not then P $\subseteq$ M[x] for some maximal ideal M of D, and by going up in the integral extension D[x] $\subseteq$ D′[x], there is an upper to zero P′ in D′[x] and a prime ideal M′ of D′ with P′ $\cap$ D[x] = P, M′ $\cap$ D = M, and P′ $\subseteq$ M′[x]. By [6, Theorem 19.15] this is impossible. Therefore, c(P) = D, as claimed. In particular, $c(P)_t$ = D, and by Theorem 1.4 P is a maximal t-ideal of D[x]. Hence D is a UMT-domain.

COROLLARY 3.6. Let D be a one-dimensional domain. Then D is a UMT-domain $\Leftrightarrow$ the integral closure D′ of D is a Prüfer domain.

Proof: This follows easily from Theorem 3.5 and the fact that height one primes are t-ideals.

Our final result in this section characterizes Noetherian UMT-domains.

THEOREM 3.7. A Noetherian domain D is a UMT-domain $\Leftrightarrow$ every prime t-ideal of D has height one.

Proof: Suppose that D is a Noetherian UMT-domain, and let P be prime in D with ht P > 1. Pick a,b $\in$ P such that (a,b) is contained in no height one prime of D, and shrink P[x] to a prime Q minimal over ax − b; necessarily Q will be an upper to 0. As D is a UMT-domain, Q is a maximal t-ideal, whence P is not a t-ideal.

For the converse, let Q be an upper to 0. Then Q $\nsubseteq$ P[x] for all height one primes P, whence c(Q) $\nsubseteq$ P for all prime t-ideals P. It follows that $c(Q)_t$ = D. Hence D is a UMT-domain.

REMARK 3.8. It is well known that in a Noetherian domain every prime t-ideal is in fact an associated prime of a principal ideal. Thus a Noetherian domain is a UMT-domain $\Leftrightarrow$ every associated prime of a principal ideal has height one.

QUESTION: If D is a Noetherian domain, does it necessarily follow that there is a Noetherian UMT-domain between D and the integral closure D′? Of course, as is well known, D′ itself is a Krull domain, hence also a UMT-domain.

Section 4. A generalization of Theorem 1.4.

As a precursor to our main result, we prove the following special case.

PROPOSITION 4.1. If B is an ideal of D[x] with $c(B)_t = D$, then B is t-invertible.

Proof: Pick $g \in B$ with $c(g)_v = D$. Also, choose $f \in B$ such that $BK[x] = fK[x]$. As in the proof of Corollary 1.5, we can show that $B \subseteq (f,g)_v$. Hence if a nonzero element a of D is chosen so that $ag \in (f)$, we have $aB \subseteq (f)$. Therefore, $\frac{a}{f} \in B^{-1}$, and $a = \frac{a}{f}f \in BB^{-1}$. Since $(a,g)_v = D[x]$ and $(a,g) \subseteq BB^{-1}$, this implies that $(BB^{-1})_t = D[x]$.

LEMMA 4.2. If I is a fractional ideal of D and A is an ideal of D[x], then $c(IA) = Ic(A)$.

Proof: That $c(IA) \supseteq Ic(A)$ is clear. Let $d \in c(IA)$. Then $\exists\, u_1,\dots,u_k \in I$ and $f_1,\dots,f_k \in A$ such that $d \in c(\sum_{j=1}^{k} u_j f_j) \subseteq \sum_{j=1}^{k} u_j c(f_j)$. Since each $u_j c(f_j) \subseteq Ic(A)$, we have $d \in Ic(A)$, completing the proof.

THEOREM 4.3. Let B be an ideal of D[x]. If c(B) is t-invertible, then B is t-invertible.

Proof: Consider the ideal $c(B)^{-1}B$. By Lemma 4.2, $c(c(B)^{-1}B) = c(B)^{-1}c(B)$. Hence $c(c(B)^{-1}B)_t = D$, whence by Proposition 4.1, $c(B)^{-1}B$ is t-invertible. It follows easily that B is t-invertible.

The converse of Theorem 4.3 is far from true, as we show in Theorem 4.5 below.

LEMMA 4.4. If B is an ideal of D[x], then $c(B_t)_t = c(B)_t$.

Proof: $B \subseteq c(B)[x] \Rightarrow B_t \subseteq (c(B)[x])_t = c(B)_t[x] \Rightarrow c(B_t) \subseteq c(B)_t$. The result follows easily.

THEOREM 4.5. D is a PVMD $\Leftrightarrow$ for all ideals B of D[x], t-invertibliliy of B implies t-invertibility of c(B).

Proof: $\Leftarrow$) Let I be a finitely generated ideal of D, and choose $f \in D[x]$ with $c(f) = I$. Clearly, $fD[x]$ is (t-) invertible, whence $I = c(fD[x])$ is t-invertible. Therefore, D is a PVMD.

$\Rightarrow$) Now let B be a t-invertible ideal of D[x]. By Proposition 2.6, $B_t = gA[x]$, where $g \in D[x]$ and A is a t-invertible ideal of D. By Lemma 4.2 $c(B_t) = Ac(g)$. Since D is a PVMD, c(g) is t-invertible, whence so is $c(B_t)$. By Lemma 4.4 $c(B)_t = c(B_t)_t$. Therefore, c(B) is t-invertible.

An argument similar to, but easier than, that for Theorem 4.5 yields the following characterization of Prüfer domains.

THEOREM 4.6. D is a Prüfer domain $\Leftrightarrow$ for every ideal B of D[x], invertibility of B implies invertibility of c(B).

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