Numbers, large and small; uses and misuses

Muhammad Zafrullah

Let me start with the concept. A number can, for now, be considered a label that we assign to a quantity with reference to a certain standard (unit etc.). One of the two numbers could be larger than another if the quantity that it represents is larger than the other with reference to the same standard (or units).

Equality of two numbers can also be visualized in the same manner. These days Mathematicians draw a number line by marking unit lengths on a line and claiming with proof that every (real number) lies on this line. A number without any qualification is what is known as a real number, to Mathematicians.

The fact is that every length can be marked on the real line and so every length represents a number on a fixed real line. Numbers have an important place in Mathematics.

Commenting on the nature of Mathematics, a great Mathematician by the name of Kronecker (1823-1891) once said, “God made integers; all else is the work of man”. (Of course he did not say it in English, but I will keep the quotes all the same.) There also goes a saying, again attributed to Kronecker that God made numbers and man, made Mathematics.

I have often wondered: Was it really integers that God made, or was it the sense of quantitative measure and competitiveness that He bestowed on Man? In fact I have the feeling that God made everything and Man, made numbers to count his blessings, measure his goodies and compare them with others’.

You see my trouble is that according to history the concept of number, even that of the integers was developed very slowly and I dare say laboriously, by humans. My Pakistani (possibly) great-great…great granduncle Panini was, according to some, the first human to use letters of alphabet for natural numbers (lived somewhere between 8^{th} century BC to 4^{th} century BC).

Later came the nine digits of Hindu Numerals that were made into Hindu-Arabic Numerals with the addition of a symbol for zero. Let's also not forget the earlier civilizations such as the Babylonians who had a, good, working, though laborious, system of sixties; the sexagecimal system.

In fact every known ancient civilization had to develop some concept of counting numbers and fractions to have its name in history and most of them had the base 10 system as the Arabs, the Chinese, the Egyptians and the Hindus.

I do realize that some of my readers may not believe in God. So, I refrain from delving too deep into who made what. But, whoever made them, numbers are very important in our lives. A young child is very interested in how tall she/he has become and wishes to keep count of feet, inches etc. or centimeters.

Then at school we are interested in grades which of course depend on the marks obtained in examinations. As we grow up a little more the notion of wealth takes control and wealth as you know has to be measured or counted to give us the "imagined" edge that we often crave.

Numbers have uses other than the simple minded ones that I have just alluded to. We add them, multiply them and take their means. Means are something like n players pooling together their winnings and dividing the sum by n. Of course, taking average does not give any comfort to the loser but people take averages anyway. Let me put it this way, I come from a country with very high per capita income but I am a very poor man.

The mention of a mean often reminds me of an old Hindu joke. It is said that a Hindu Pundit was traveling with his family, on foot as they did in the old days. On their way they had to cross a river. Only the Pundit knew how to swim and naturally he was worried about the safety of his family.

So he went into the water to gauge its depth at various points on the intended path. Then he came back to his family, did the calculation and found out that on average the water was knee deep for his youngest child. In went the family and at the first deep part everyone except the Pundit drowned.

The Pundit, surprised, came ashore, did the calculation again and exclaimed: My calculation works fine then what drowned the family of mine? This joke is not intended as an insult to anyone, it is in this way the old sages told us to be wary of depending on averages blindly. (In ancient India Pundits were the only ones who knew “sophisticated Arithmetic”.)

Sometimes averages tell the true story, but that is when they get suppressed. For example if the number of billionaires in a country increases it may not indicate prosperity of the country; but politicians will use such information to beguile their audience anyway.

There are other numbers, produced by Statisticians and Mathematicians that the politicians and the economists use. But remember the politicians would use those numbers to increase their influence and the academics will use them to support their theories. The truth comes out when there is a catastrophic market crash, as it happened towards the end of the G. W. Bush presidency.

Catastrophic market crashes remind me of the Catastrophe theory by Rene Thom. According to Rene Thom there are (dynamical) systems, with markets included, that would, at some point, give abrupt (catastrophic) outputs in response to continuous inputs. (Catastrophic events are often studied by economists, Market analysts and Engineers.)

More pleasant catastrophic events are the buds bursting to become flowers. (As the time passes the bud becomes larger and larger on the continuous supply of juices from the plant, but there comes a time the bud can remain bud no longer.) On the other hand the light switches going from “on” to “off” and vice versa represent useful catastrophic events. In a minute I will be talking about some unpleasant (neo-) catastrophic events.

Next, there are large numbers, there are small numbers, and there are negligible numbers. They play their roles in our lives, often mysteriously. A lot of small numbers can add up to an unmanageably large number. Entropy of a system can increase as a result of apparently insignificant heat exchanges. Some inefficient

internal combustion engines that seemed to cause benign or insignificant pollution are now threatening our environment.

The trouble is that even though a lot of mass got created at the big bang and a lot of energy came out from presumed nothingness we cannot, using ordinary tricks, create, destroy or ignore them.

I have only mentioned one aspect of pollution; there are other kinds of pollution. Carelessly discarding or spraying apparently insignificant amounts of harmful chemicals in the past is playing havoc on our environment. Some fish and amphibian creatures are having sex ambiguity problems. Dumping waste, industrial or urban into the rivers is killing the oceans. Also, making small savings on essential maintenance of an oil rig can cause big oil spills, as we are witnessing in the Gulf of Mexico.

Big scientists are working on these problems. All I can do is consider an example and draw some conclusions.

Fire in the compost is an unwanted hazard in manure (organic fertilizer) factories. The reason for the fire is that when the animal and/or human waste and other biodegradable materials are piled together there are chemical reactions that cause heat. The continuous increase of heat energy causes the temperature to go so high that the material catches fire.

Fire in the compost heap is an undesirable event. So, the factory management hires someone to monitor the compost heap and keep its temperature from going beyond a certain limit by turning the compost heap. In my opinion this offers a clue to the management of systems that may have undesirable catastrophic events. The clues are: Vigilance, checks and balances, and maintenance.

Here's another example of accumulation of small quantities causing big problems. Small amounts of heat and moisture emanating from various buildings in a large city give the city its own climate. Some cities have had it so bad that landing a plane at their airports is becoming trickier and trickier. It might be of interest to compare the climates of large cities with (a) the climates of their surrounding areas and (b) with their climate a century ago.

On the other hand large numbers have their own charm. Some large numbers are regarded with respect and some with fear. A country with high GNP (gross national product) is considered rich, a country with a large population is considered big, an earthquake high on the Richter scale is feared and so is a big tsunami.

We may note however that the standards of “large”, “small” and insignificant may at times be misleading. An earthquake high on the Richter scale in a desert may even go unnoticed whereas a moderate looking earthquake, in the vicinity of 6.5, may well turn out to be deadly if it takes place in a city with a large population and no recent earthquakes. The devastation, in terms of human lives, caused by a tsunami, can be averted with an efficient early warning system.

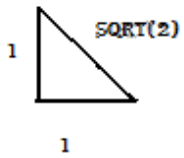
On the other hand a country with a small population can beat the daylights out of a population-wise large country with the help of its technology and economy, or with the help of some powerful friends.

Oh and please do not get me wrong. Numbers often serve as good indicators. All I am saying is that those indicators, when misused or misunderstood, can cause a lot of headache.

As human needs grew, different kinds of numbers came into being. In the beginning out of necessity, numbers such as fractions (integer divided by nonzero integer) and later out of curiosity, such as irrational numbers and out of Mathematical necessity such as complex numbers. The real numbers turned out to be the collection of rational and irrational numbers. The imaginary numbers and the complex numbers are the ones that entertain the square root of -1.

It seems to me that fractions appeared even before the concept of number took shape. Well if a mother had two children and one loaf of bread she would have to halve it to feed the two hungry mouths. The Greeks were very fond of fractions and considered them the only rational form of numbers.

But then Pythagoras proved the Pythagoras Theorem, apparently, hundreds of years after the Chinese: If we add the square of the base of a right triangle with the square of the height we get the square of the hypotenuse, the third side, of the triangle. An adherent of Pythagoras once considered a right triangle with base = 1 and height =1. By rights the length of the third side of the triangle should be $\sqrt{2}$. But, $\sqrt{2}$ could not be expressed as a fraction, it is a proven fact.)



So$\sqrt{2}$ (or sqrt (2)) and numbers that could not be expressed as fractions had to be called irrational, otherwise there seems to be no irrationality around the irrational numbers. The Pythagoreans tried to hide irrational numbers, for a while, but the trouble is that a lot of useful numbers such as π (Pi), the ratio between the circumference and the diameter of a circle, and e the "natural base" are irrational.

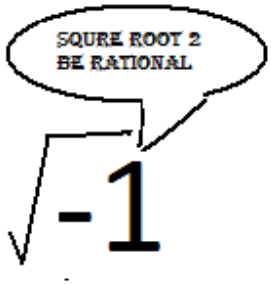
The natural base came out of Mathematical necessity. Without the natural base you cannot do a lot of the Calculus, without π you cannot do trigonometry and without these two numbers your science and technology grinds to a halt.

One use of the irrational numbers, from a Mathematician's point of view, is the amount of Mathematics that was created to prove the irrationality of Pi and e etc. The misuses of irrational numbers may often be found in their estimation as decimal fractions.

The imaginary number came out of Mathematical necessity but any attempt at explanation, at this point, will make this article too heavy. So I end their mention with the following cartoon which my Math-hating Chemist son has recently sent me. The cartoon goes like this:

$\sqrt{2}$ tells iota =$\sqrt{-1}$ to get real and iota in response tells $\sqrt{2}$ to be rational.

The square of $\sqrt{-1}$ is -1, a negative number and it is well known that the square of a real number must always be nonnegative, so iota cannot be a real number. The pun here is in the dual meanings of “get real” and “be rational”.



I must say that like a lot of things, numbers are not good or bad in themselves it is their use or misuse that makes them look good or bad. Those, such as Wall Street wizards, who play with numbers to give false impressions and false hopes, may deserve all the wrath of Saint Augustine. Saint Augustine; by the way, was an old sage (born in the fourth and died in the 5th century AD) who was dead against Mathematicians as the following quote indicates.

"The good Christian should beware of mathematicians, and all those who make empty prophecies. The danger already exists that the mathematicians have made a covenant with the devil to darken the spirit and to confine man in the bonds of Hell" (St. Augustine, *De Genesi ad Litteram*, Book II, xviii, 37).

Finally, and it must be said. Some folks think that by “Mathematicians” Saint Augustine meant Astrologers and it is plausible, but Astrologers did play with numbers. Also, probably Kronecker meant that taking integers as given, one can build Mathematics. He was probably referring to Dedekind’s work on defining real numbers from rational numbers, using his now famous Dedekind cuts. But then he should have said that.