# MATH 360 Test 03, Spring 2002

Name:
Show all work and attach necessary printouts for computer-work.

1. (a) Solve the following differential equation: $((t-1)^2-1)\dfrac{d^2y}{dt^2}+5(t-1)\dfrac{dy}{dt}+3y=0$ by means of a power series about $t_0=1$. Find the recurrence relation; also find the first four terms in each of two linearly independent solutions. (b) Use a suitable Maple command to find the solutions required above.
2. Solve the differential equation $\dfrac{d^2y}{dt^2}-2\dfrac{dy}{dt}+y=0$ by means of a power series about $t_0=0$. Give complete series representations of the two linearly independent solutions.
3. (a) Solve by hand, showing all work, the initial value problem:

$t^2y''+ty'+4y=0, \quad y(1)=-2, \quad y'(1)=2$. (b) Discuss the behavior of the solution close to the point $t=0$. (c). To support your discussion in (b) use plot command to plot the graphs of the solution over the intervals (0,0.5], (0,0.1], (0,0.01].

4. Rewrite $h(t)=\begin{cases}0 & 0\le t<10\\ t-5 & 10\le t<20\\ 1 & t\ge 20\end{cases}$ using Heaviside step functions.
5. (a) Find the Laplace transform using the definition, showing all work, and verify your results by using a suitable Maple command.

(i). $e^{3t}\sin 2t+3t^3$ (ii) $2t\cosh(2t)-7t^2e^{0.5t}+e$ (iii) $u_3(t)(t+1)$

(iv) $h(t)=\begin{cases}0 & 0\le t<10\\ t-5 & 10\le t<20\\ 1 & t\ge 20\end{cases}$

6. (b). Find the inverse Laplace transform using the standard techniques, showing all relevant references, and verify your results using a suitable Maple command.

(iv) $\dfrac{e^{-s}}{s}+\left(\dfrac{s}{s^2-4s+9}\right)+\dfrac{5}{s}$

(v) $\dfrac{3!}{(s-2)^3}$

7. Find the solution of the initial value problem:
$y''+2y'+y=2(t-3)u_3(t), \quad y(0)=2, \quad y'(0)=1$.
8. . Solve the initial value problem using the method of Laplace transform, showing all work: $y''+y=\sin t, \quad y(0)=1, \quad y'(0)=2$.