Some examples of determining a suitable form for the particular solution *Y(t).* I have taken up the approach of splitting the non-homogeneous part to make simpler non-homogeneous equations and then putting all the possible particular solutions together. These problems have been taken from page 178 of your book.

19. $y'' + 3y' = 2t^4 + t^2\exp(-3t) + \sin(3t)$

Corresp. Homog. Eqn.: $y'' + 3y' = 0$

Char. Eqn.: $r^2 + 3r = 0$ ; r = 0; r = -3

Clearly $y = c$ and exp(-3t) are solutions.

$y_c = c + d\exp(-3t)$

Take $y''+3y' = 2t^4$ , $y_1 = t(A_o t^4 + A_1 t^3 + A_2 t^2 + A_3 t + A_4)$ (because y= c is a solution.)

Take $y''+3y' = t^2\exp(-3t)$ , $y_2 = t(B_o t^2 + B_1 t + B_2)\exp(-3t)$ (because exp(-3t) is a solution)

Take $y''+3y' = \sin(3t)$, $y_3 = A\sin(3t) + B\cos(3t)$ (because there is no Complex solution)

The possible particular solution is: $Y = y_1 + y_2 + y_3$ and the general solution is

$y = y_c + Y =$

20. $y''+y = t(1+\sin(t)) = t + t\sin t$

Homog. Eqn.: $y'' + y = 0$ . Char. Eqn.: $r^2 + 1 = 0$, $r = \pm i$, $y_c = C_1\cos t + C_2\sin t$

Take $y''+y = t$, $y_1 = A_o t + A_1$ (Since $y = t$ is not a solution of homog. Eqn.)

Take $y''+y = t\sin t$, $y_2 = t(A_o t + A_1)\sin t + t(\cot t + C_1)\cos t$ (Since $\exp(it)$ is a solution.)

So $Y = y_1 + y_2$ and $y = Y + C_1\sin t + C_2 Cost$

21. $y''-5y'+6y = \exp(t)\cos(2t) + \exp(2t)(3t+4)\sin t$

Homog. Eqn.: $y''-5y'+6y = 0$ . Char. Eqn.: $r^2 - 5r + 6 = (r-3)(r-2) = 0$, *r = 2, r = 3.*

So $y_c(t) = C_1\exp(3t) + C_2\exp(2t)$

Take $y''-5y+6y = \exp(t)\cos 2t$ , $y_1 = A\exp(t)\cos(2t) + B\exp(t)\sin(2t)$ (Because $r \neq 1+2i$ )

Take $y''-5y+6y = \exp(2t)(3t+4)\sin t$

$y_2 = \exp(2t)(Ct+D)\sin t + \exp(2t)(Et+F)\cos t$ (Because $r \neq 2 \pm i$ )

Combining, $Y = y_1 + y_2$ and the general solution is

$y = y_c + Y$

22. $y''+2y'+2y=\exp(-t)(3+2\cos t+4t^2\sin t)$

Homog. Eqn.: $y^{//}+2y^{/}+2y=0$. Char. Eqn.: $r^2+2r+2=0$, $r=\dfrac{-1\pm 2i}{2}=-\tfrac{1}{2}\pm i$

So $y_c=\exp(-\tfrac{1}{2}t)(A\cos t+B\sin t)$. Take

$y''+2y'+2y=3\exp(-t)$

$y''+2y'+2y=2\exp(-t)\cos t$

$y''+2y'+2y=2\exp(-t)t^2\sin t$

Since none of the functions on the right is a solution of the homog. Eqn. we get

$y_1=A\exp(-t)$

$y_2=tB\exp(-t)\cos t+C\exp(-t)\sin t$

$y_3=t\exp(-t)(D_o t^2+D_1 t+D_2)\cos t+t\exp(-t)\,(E_o t^2+E_1 t+E_2)\sin t$

$Y=y_1+y_3$, because $y_2$ is included in $y_3$.

23. $y''-4y'+4y=2t^2+4t\exp(2t)+t\sin(2t)$

Homog. Eqn.: $y''-4y'+4y=0$. Char. Eqn.: $r^2-4r+4=0=(r-2)^2$, r = 2 repeated twice

$y_c=(A+tB)e^{2t}$

Take: $y''-4y'+4y=2t^2$, $y_1=A_0t^2+A_1t+A_2$ (r = 0 is not a solution of homog. eqn.)

Take: $y''-4y'+4y=4t\exp(2t)$, $y_2=t^2(B_0t+B_1)\exp(2t)$ (because $e^{2t}$ is a twice repeated solution of the homogeneous equation)

$y''-4y'+4y=\sin(2t)$, $y_3=(C_0t+C_1)\sin(2T)+(D_0t+D_1)\cos(2t)$ ( because neither sin (2t) nor cos (2t) is a solution of the homogeneous equation).

So, the possible particular solution is $Y=y_1+y_2+y_3$.

$$(\cos t + C_1)\sin(2t) + (D_0 t + D_1)\cos(2t)$$