QUESTION: (HD0807) In Cohn's paper "Bezout rings and their subrings" theorem 2.4 has no proof, and I was able neither to find any source of it, nor build it by myself. You may refer me to other material or internet.

ANSWER: Theorem 2.4 of Cohn's paper [Proc. Cambridge Philos. Soc. 64(1968), 251-264] says: Every HCF-ring is a Schreier ring. This in my usual terminology means that a GCD domain has the Schreier property. A direct method of proof is to use the GCD property to prove the existence of the Schreier property. Now we know that a GCD domain is integrally closed and a Schreier domain is an integrally closed domain in which every nonzero element is primal. So all we need to show is that every nonzero element of a GCD domain D is primal. For this let x be any element of a GCD domain D and suppose that $x \mid yz$, in D, for some $y, z \in D\backslash\{0\}$. Let $d = GCD(x, y)$. Then $x = x_1 d$ and $y = y_1 d$ where $GCD(x_1, y_1) = 1$. So $x \mid yz$ can be rewritten as: $x_1 d \mid y_1 dz$ cancelling out d we have $x_1 \mid y_1 z$, but then $x_1 \mid z$, because $GCD(x_1, y_1) = 1$. So $x = x_1 d$ where $d \mid y$ and $x_1 \mid z$. Since y, z are arbitrary such that $x \mid yz$, x is primal and since x is arbitrary we conclude that every element of the GCD domain D is primal.

Note: I was surprised that you could not find any hint to the proof of GCD $\Rightarrow$ Schreier, with all the interest in pre-Schreier domains. So I asked a guru of sci.math and got the following references: (This person goes by the name of Bill Dubuque.)

[1] http://google.com/groups?selm=y8z7k9leoku.fsf%40nestle.ai.mit.edu
http://google.com/groups?selm=y8zhc9824he.fsf%40nestle.csail.mit.edu
[2] http://google.com/groups?selm=y8zr658xqm9.fsf%40nestle.csail.mit.edu
[3] http://google.com/groups?selm=y8z8yto8k85.fsf%40nestle.ai.mit.edu