QUESTION: (HD0313). Is every essential domain a P-domain.?

ANSWER: No. There is at least one known example of an essential domain which is not a P-domain. This example was produced by William Heinzer [An essential domain with a non-essential localization, Can. J. Math. 38(2) (1981) 400-403].

For a reader who is not familiar with the terminology, the above answer could raise a lot of other questions. Some of those questions are answered below.

Let D be an integral domain. Call a prime ideal P of D essential if the localization D_P is a valuation domain. Call a family of prime ideals $\mathcal{F} = \{P_\alpha\}_{\alpha \in I}$ a defining family of prime ideals for D if $D = \cap_\alpha D_{P_\alpha}$. We call D essential if D has a defining family of essential primes. In the Mott-Zafrullah paper on Prufer v-multiplication domains [Manuscripta Math. 35(1981), 1-26] a domain D was called a P-domain if D is essential and every quotient ring of D is essential and it was shown that even a P-domain may not be a Prufer v-multiplication domain. (Recall that D is a Prufer v-multiplication domain (PVMD) if and only if D_P is a valuation domain for each maximal t-ideal P of D, read HD0302 for maximal t-ideals). To give an example, Mott and Zafrullah showed that the example of an essential non-PVMD produced by Heinzer and Ohm in [Canad J. Math.25(1973), 856-861] was in fact a P-domain. This led to the question: Must all essential domains be P-domains? This question was answered in the negative by Heinzer in [Can. J. Math. 38(2) (1981) 400-403] mentioned above.