QUESTION: (HD 1204) In the abstract of a paper that appeared in [JPAA, 214(9) (2010), 1633-1641] D'Anna, Finocchiaro and Fontana mention as classical the constructions A+XB[X], A+XB[[X]] and D+M. Where can I learn about these constructions?

Answer: The best way should be to look up the references at the end but, apparently, while for the other "classical" constructions the sources are adequately indicated I could not find any reference to the above mentioned constructions, except for a 2001 paper by Dobbs and Khalis on the $A+XB[[X]]$ construction and one on the Krull and valuative dimensions of $A+XB[[X]]$ construction, hardly representative. In any case you can look up my survey on this topic: Various facets of rings between $D[X]$ and $K[X]$ [Comm. Alg. 31(5) (2003), 2497-2540]. This paper though essentially addresses the $A+XB[X]$ construction it gives references to the important sources on the $A+XB[[X]]$ construction (Dumitrescu, Salihi, Radu, Shah [Comm. Algebra 28(3) (2000), 1125-1139]) and on the general $D+M$ construction (Brewer and Rutter [Michigan Math J. 23(1) (1976), 33-42]). Fontana should remember this paper well. This paper part appeared, with a funny title, in ["Commutative Ring Theory and Applications" Volume 231 (2003), Dekker Lecture Notes series] and Fontana was an Editor.

To make sure that you learn something more than just references, let me also mention that the $D+M$ constructions were initially studied by Bastida and Gilmer [Michigan Math. J. 20 (1973), 79-95] in a somewhat restricted set up. The basic set up of Bastida and Gilmer was: Let V be a valuation domain expressible as $K+M$, where K is a field and M is the maximal ideal of V, and let D be a subring of K. Then $R=D+M$ is a subring, of V, called the $D+M$ construction. (An easy example of such a V is $K[[X]]=K+XK[[X]]$.) On the other hand the general $D+M$ construction of Brewer and Rutter goes as: Let R be a domain such that R can be written as $R=K+M$ where K is a field and M is a maximal ideal of R then for a subring D of K the ring $D+M$ is dubbed as the general $D+M$ construction. (A prototype of such an R is $R=K[X]=K+XK[X]$.)

While I am at it, let me also describe the other two constructions. Let B be an integral domain, A a subring of B and let X be an indeterminate over B. Then $A+XB[X]=\{f\in B[X]:f(0)\in A\}$. To my knowledge this type of constructions first appeared as the $D+XD_S[X]$ construction in a paper of mine with Costa and Mott [J. Algebra 53(1978), 423-439] and the more general $A+XB[X]$ in a paper of mine with Dan and David Anderson [Houston J. Math. 17(1) (1991), 109-129]. Next, likewise, $A+XB[[X]]=\{f\in B[[X]]:f(0)\in A\}$, with A,B,X described as above. About $A+XB[[X]]$ construction I do not know much except that Mohammed Khalis wrote a thesis, in French, on these constructions in 2001 at University of Ben Abdellah, Fez Morocco.