QUESTION: (HD1208) Let R be a Noetherian local domain and let P be a height one prime ideal of R. Can we find an element $x \in P$ such that P is the only minimal

prime containing x.

ANSWER: Not always. Here's a way of showing that. Suppose on the contrary that for each height one prime P of a Noetherian local domain R we can find x such that P is the only minimal prime of x. Let us also take our Noetherian local domain to be integrally closed, then R is a Krull domain and P being the only minimal prime of some $x \in P$ means $(x) = P^{(n)}$.To see the validity of the previous statement note that D is a Krull domain if and only if every proper principal ideal (x) of D has a primary decomposition of the form $(x) = P_1^{(n_1)} \cap P_2^{(n_2)} \cap ... \cap P_r^{(n_r)}$ where P_i are height one primes of D (see [AMZ, Corollary 3.2]). Thus by our assumption each height one prime P of the local Krull domain R is such that $(x) = P^{(n)}$ for some $x \in P$. But this means that $Cl(R)$, the divisor class group of R, is torsion [1]. So if we can find an integrally closed Noetherian local domain whose class group is not torsion then we have the required contradiction. But, on page 67 of R. Fossum's book [F] there is an example of a Noetherian local Krull domain A with divisor class group equal to Z, the set of integers. As Z is torsion free, for no non-principal height one prime P and for no positive integer n can we have $P^{(n)}$ principal. Thus we have an integrally closed Noetherian local domain such that for for every non-principal height one prime P there is no $x \in P$ with P the only minimal prime of x.

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[F] R. Fossum, The divisor class group of a Krull domain, Ergebnisse der Mathematik und ihrer grenzgebiete B. 74, Springer-Verlag, Berlin, Heidelberg, New York, 1973.