QUESTION (HD 1801): In your paper on $*$-super potent domains at https://arxiv.org/abs/1712.06725, you define a $*$-super rigid ideal I requiring that I is contained in a unique maximal $*$-ideal M and that F is $*$-invertible for every finitely generated ideal $F \supseteq I$. Looking at the proof of part (3) of Theorem 1.11 it seems that in the definition of a super rigid ideal, above, you seem to allow F to be a fractional ideal. Is that necessarily the case? (Professor D.D. Anderson put that question to me.)

ANSWER: It looks like that! But F ain't a fractional ideal. Here's how.

Let's be clearer and say that in the definition of a $*$-super rigid ideal I, the ideal F must always be integral. Now let I and J be two $*$-super rigid ideals contained in the same maximal $*$-ideal M. To show that IJ is $*$-super rigid we must show that F is $*$-invertible for each finitely generated integral ideal $F \supseteq IJ$. If $F^* \supseteq I$ or J then F is $*$-invertible, right off the bat. So let's assume that F does not contain, say, I. Claim: $F^* = D$ or $F \subseteq I^*$. If $F^* = D$ then F is $*$-invertible anyway. So let's assume that $F^* \neq D$. Indeed as $F \supseteq IJ$, and F is integral, $F \subseteq M$, because M is the only maximal $*$-ideal containing IJ and so $(F + I)^* \neq D \neq (F + J)^*$. Since $F + I$ contains the $*$-super rigid I, $F + I$ is $*$-invertible. Let $(F + I)^* = K$ and so $(FK^{-1} + IK^{-1})^* = D$. Since both $F, I \subseteq K$ we have $(FK^{-1})^*, (IK^{-1})^* \subseteq D$. Indeed as F^* does not contain I, $(FK^{-1})^* \neq D$ and so $(FK^{-1})^* \subseteq M$. Now $(IK^{-1})^*$ cannot be in M and hence in any maximal $*$-ideal N. For if $(IK^{-1})^* \subseteq M$, then $(FK^{-1} + IK^{-1})^* \neq D$ a contradiction and if $(IK^{-1})^* \subseteq N$ for any other maximal $*$-ideal N then $I \subseteq N$ a contradiction to the fact that M is the unique maximal $*$-ideal containing I. Thus we conclude that $I^* = K$, forcing $F \subseteq I^*$. Now $F \supseteq IJ$ implies $FI^{-1} \supseteq J$ (and FI^{-1} is integral). Now, since $*$ is of finite type, $I^{-1} = A^*$ for some finitely generated A. So $(FA)^* \supseteq (IJA)^* = J^* \supseteq J$. Again since $*$ is of finite type we can arrange for a finitely generated $H \subseteq (FA)^*$ with $H \supseteq J$ and $H^* = (FA)^*$ we conclude that $(FA)^*$ is $*$-invertible, because H is $*$-invertible. But then F is $*$-invertible.

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