QUESTION (HD 2002) I have been reading your paper, "On $*$-homogeneous ideals" https://arxiv.org/pdf/1907.04384.pdf

Boy! What a mistake to make! I read your "explanation" after Theorem 2.3. Though I could not pinpoint the mistake , but isn't your admission-like explanation proof that you made a huge mistake?

ANSWER: People do make mistakes. When I was a reviewer for Math Reviews, I pointed out quite a few of them, including one of my own. (In any case there are a lot of mentions of mistakes in Mathematical literature. So if I made a mistake or two, no big deal.) As a referee I am considered helpful but hard, I do make helpful comments but I am very exacting if I come across mistakes and sloppy language. I also caught some folks trying to get away with pseudo Math and caught a bunch of guys trying their hand at plagiarism. In ordinary life I am a tit-for-tat person and I won't let anyone push me around because I have a foreign face. In short, I have been miserable ever since I got to the US. For that's when all the above-mentioned activities became intense. I have lost jobs, I have lost health and I have lost a lot of friends. (However hard you try people usually know who's the referee and reviewers have their names printed!) So if there's a mistake or omission in my work some folks would like to bury me so deep that I never make it back to the surface. It has happened to me a couple times before, and I survived.

Now, coming to my mistake. I wrote, in the proof.

"Thus by Zorn's Lemma, $\mathcal{S}$ must have a maximal element $U = \{V_1, V_2, ..., V_n\}$.

That each of V_i is homogeneous follows from the observation that if any of the V_i say V_n by a relabeling, is nonhomogeneous then by Lemma 2 V_n is contained in at least two v-comaximal elements which by dint of containing V_n are v-comaximal with $V_1, ...V_{n-1}$. This contradicts the maximality of U." (The mistake, as it was pointed out to me, that even though still in $\mathcal{S}$, the resulting set isn't comparable with U.)

I could have said "Thus by Zorn's Lemma, $\mathcal{S}$ must have a maximal element $U = \{V_1, V_2, ..., V_n\}$ of largest size.

That each of V_i is homogeneous follows from the observation that if any of the V_i say V_n by a relabeling, is nonhomogeneous then, by Lemma 2, V_n is contained in at least two v-comaximal elements which by dint of containing V_n are v-comaximal with $V_1, ...V_{n-1}$. This contradicts the maximality of the size of U."

(Now how can we pick an element U of maximal size n? Easy. Look at $L = \{U \in \mathcal{S} \mid U \text{ is maximal}\}$. Now take the set $M = \{|U|\}$. Then M has a minimum and a maximum, minimum because M is a set of integers and maximum because of the condition that every finitely generated nonzero ideal is contained in at most a finite number of mutually v-cmaximal elements.)"

Now believe it or not. When I fixed n as the size of U, I was assuming it was the largest size and that I was getting the contradiction on the size. Call it my dyslexia or call it my misfortune, or call it Tiberiu Dumitrescu's fortune, I will come to that in a minute, this is how it happened. (He had given half a proof and insisted that it should work. When I challenged him he did not give me a satisfactory explanation. Then there was pressure from him for us to publish this paper quicker and I do not work well under pressure.)

I realized soon enough that the proof won't stand the scrutiny, gave the above proof to form a corrigendum and then gave the proof that was eventually included in Chang and Hamdi's paper [1]. But how do you write a Corrigendum without the other author? So I resorted to other proofs that I have indicated in the paper you mention.

Having said all that, let's see what benefits I drew from my mistake.

(1) I found out what Tiberiu Dumitrescu was made of. (I had already noticed that he'd rush things in and then, when the galley prints were in, he'd send in his corrections, leaving me to explain to the editors. His overcleverness has made some other folks unhappy too. I know of at least two highly placed, Mathematician who almost came to blows in emails, because of Tiberiu's tricks.)

(2) The way Tiberiu Dumitrescu, and generally some Romanians, treated me, reminded me of Vlad the Impaler, who carved his name in history by impaling hundreds of (Muslim) Turks. That murderer, by the way, is a Romanian hero. I have had my webpage hacked and I have been chased around on an internet site with silly alternatives to my suggestions and solutions of problems. A Romanian sub editor of a journal sent me a longish paper to referee. I spent three months reading and writing notes to help the authors with their English and occasionally with topic at hand. Of course the paper was good, or I would have rejected it right at the start. Now as I send in the paper with the recommendation to publish it after the changes are made the fellow comes back asking me to reject it, citing backlog etc.. At a weak moment I decided to comply. But that lost me an old friend. (I have told you that it's hard to hide yourself, especially if you are making comments.)

Now, my problem is that, I am a member of the Ahmadiyya Community in Islam. My community has been declared non-Muslim in Pakistan. Moreover, in the eyes of many a Pakistani, I am a non-Muslim and, by the belief of some of them, worthy of being killed on sight. I was under pressure from some Pakistanis to visit their schools and give talks and/or teach. But my children were concerned that I could well be killed. (Most adamant was my oldest son who'd met Tiberiu when Tiberiu visited me in Fayetteville Ark, and stayed at our apartment.) In any case when the word got out that I had "made a mistake" the invitations and requests died out. (This happened long before that 2019 Chang and Hamdi paper [1]!) Well it ain't a big loss, Pakistan is the country where my B.Sc. (Honours) result was announced "to be declared later on" and I was told to apply for a "Pass Course degree" instead, because the result could stay "to be declared later on", indefinitely. Tiberiu now is a darling of those Pakistanis because they think Tiberiu got me. I am not too happy that Pakistanis got Tiberiu who can't be trusted. But perhaps that's fate. As far as I am concerned, I'd let my work speak for myself, even the work that went into that "blemished" paper.

(3) I have published quite a few papers with the Journal of Pure and Applied Algebra, but when the late Tony Geramita started handling my papers there started the feeling that something was not quite right. With this fateful paper, the whole thing came to a head. The referee had actually cut the paper out of

shape and with such remarks that my blood boiled. Frankly, if I were the sole author I would have said "the hell with the referee, I'd publish it elsewhere". But as Tiberiu had written the material, that I had sent him, in a specific way, that he was proud of, he was keen on publishing it in JPAA. So, we patched up whatever was left of the paper and submitted. That left me with an odd feeling. So I put the original submission on my web page as [2]. Sure enough, by the time our paper went on line there was a doctored version [4] of it on Arxiv. They published it later as [5] with the remark, that this result was proved independently in [3]. I won't go into whodunit, it's for the administration of JPAA to find out and I have a feeling they know it. I am grateful to G.W. Chang that he brought it up as in [hd1802][6] and highlighting "my mistake" established the main theorem as my theorem. (It was my theorem to start with. The proof was changed out of shape by Tiberiu, enough that I got confused. This is what he wrote in his two emails, one sent on 8-18-2009: I think we better write up a separate paper. I adapted the material for star operation (I hope this new form will amuse you). If you like you can start getting the introduction and supportive material ready. Also, we can think to add the old poset part that we have (maybe). The other on: 8-19-2009: Attached I send you a new version of the script. To what I sent previously, I added the poset stuff modeled exactly on the domain case proof (I hope you'll enjoy). I apologize if sending this too soon puts your editing work in trouble.)

(4) I have had my ups and downs and generally I was not very happy with what I got out of coming to the US. I came here as a visitor and if I had any connections back in Pakistan, I'd probably be better off. (I could have arranged some conferences there and could have edited some proceedings at least and perhaps staying for a bit in Pakistan could have helped my health.) But after this episode I am happy that I am a citizen of the United States of America. Here my name may remind some that Muslims beat the dung out of the Crusaders, but they'd still not only let me live but also help me live. My story goes like this. Early on in my career here in the US I fell ill so much that I had to leave my job and spend my time doing odd jobs, waiting to die. It turned out that since my B.Sc. (Hons.) episode I had had high blood pressure, that took its toll and my kidneys failed at a time when I had no money left on me. The state took care of me. Then I had the kidney transplant and after that there was no stopping me for a while until that whopper of a mistake was pasted on me. Now I do have gripes about how some folks have treated me, here. But, still, my heart sings "God Bless America". I am old and probably at the last rung of the ladder of life, but I am a happy man. And if I live a few years, I am sure, I will get back on my feet, by the Grace of God.

Now, in all fairness to Tiberiu, he did really well in redoing the material in a more general setting, except for a flub here and there. Here is the proof of Theorem 1 of [3], as given originally by Tiberiu in his 8-19-2009 email.

Theorem 1 *Let D a domain, $*$ be a finite character star operation on D and Γ a set of proper $*$-ideals of D. Assume that every proper $*$-finite $*$-ideal of D is contained in some member of Γ. Let I be a $*$-finite $*$-ideal of D. Then I is* *contained in an infinite number of maximal $*$-ideals iff there exists an infinite family of mutually $*$-comaximal ideals in Γ containing I.*

Call a proper $*$-finite $*$-ideal A of D *homogeneous* if A is contained in a unique maximal $*$-ideal.

Lemma 2 *Let D a domain and $*$ be a finite character star operation on D. A proper $*$-finite $*$-ideal A of D is homogeneous iff whenever B, C are proper $*$-finite $*$-ideals containing A, we get $(B, C)^* \neq D$.*

Proof. ($\Rightarrow$). Suppose that M is the only maximal $*$-ideal containing A and B, C proper $*$-finite $*$-ideals containing A. Then $B, C \subseteq M$, so $(B, C)^* \neq D$. ($\Leftarrow$). Suppose that A is contained in two distinct maximal $*$-ideals M_1, M_2. Hence $(M_1, M_2)^* = D$. Since $*$ is of finite character, we can choose finitely generated ideals $F_i \subseteq M_i$, $i = 1, 2$, such that $A \subseteq F_i^*$ and $(F_1, F_2)^* = D$. •

Proof of Theorem 1. The implication ($\Leftarrow$) is clear since a maximal $*$-ideal cannot contain two $*$-comaximal $*$-ideals. ($\Rightarrow$). Obviously, it suffices to prove the assertion for Γ = the set of all proper $*$-finite $*$-ideals of D. We prove the contrapositive statement. Assume that: ($\sharp$) there is no infinite family of mutually $*$-comaximal ideals in Γ containing I. First we show the following property: ($\sharp\sharp$) every $I' \in \Gamma$ containing I is contained in some homogeneous ideal. Deny. As I' is not homogeneous, there exist $P_1, N_1 \in \Gamma$ containing I' such that $(P_1, N_1)^* = D$ (cf. Lemma 2). Since N_1 is not homogeneous, there exist $P_2, N_2 \in \Gamma$ containing N_1 such that $(P_2, N_2)^* = D$. Note that $(P_1, P_2)^* = (P_1, N_2)^* = D$. By induction, we can construct an infinite sequence $(P_k)_{k \geq 1}$ of mutually $*$-comaximal ideals in Γ with $I' \subseteq P_k$, $k \geq 1$. This fact contradicts condition ($\sharp$). So ($\sharp\sharp$) holds. Now using ($\sharp$) and ($\sharp\sharp$) we can find a finite set $H_1, ..., H_n \in \Gamma$ of mutually $*$-comaximal homogeneous ideals containing I such that there is no $J \in \Gamma$ containing I with $(J, H_i)^* = D$, $1 \leq i \leq n$. Let M be a maximal $*$-ideal containing I and $0 \neq x \in M$. Then (I, x) is not $*$-comaximal with some ideal H_j. Since H_j is homogeneous, we get $x \in (I, x) \subseteq M_j$, where M_j the unique maximal $*$-ideal containing H_j. So $M \subseteq M_1 \cup \cdots \cup M_n$. By the Prime Avoidance Lemma, M is contained in some M_k, hence $M = M_k$. Thus $M_1, ..., M_n$ are the maximal $*$-ideals containing I.

After all the water that has flown under the bridge, this proof may make some sense. (Though, personally, I like the proof that I have given in the Arxiv paper [7] you mention. My reason: That proof brings out the true effects of Conrad's F-condition.) I knew the result was OK, but I wanted Tiberiu to defend it. So that if the referee challenged us we had something on the offer. Let me end this with a lesson for Tiberiu: You don't "demand" to be the "in-charge" of someone else's research, just because you were able to remodel some of it after learning it from him, especially if you cannot defend what you have written. (Tiberiu gradually got aggressive noting that some of his Pakistani students would rather not mention my name or my research.)

Finally, here's a public service announcement: Every time I try to look up something at Marco Fontana's homepage my anti-virus software warns me of the presence of an outbound Trojan. Some outbound Trojans can let their owners look into or even control the computers they infect. Perhaps those who have installed the Trojan on Marco's website know how my letter of 4/26/2010 to Tiberiu, got leaked (I mentioned this letter in [hd1802] [6] and [7].)

References

[1] G.W. Chang and H. Hamdi, Bazzoni's conjecture and almost Prüfer domains, Comm. Algebra, 47 (7) (2019) 2931-2940.

[2] T. Dumitrescu and M. Zafrullah, Characterizing domains of finite $*$-character https://lohar.com/researchpdf/subprufer_s.pdf

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[4] C. Finocchiaro, G. Picozza, F. Tartarone, Star-invertibility and t-finite character in integral domains, https://arxiv.org/pdf/1001.5220.pdf

[5] C. Finocchiaro, G. Picozza, F. Tartarone, (2011). Star-invertibility and t-finite character in integral domains, J. Algebra. Appl. 10:755–769.

[6] M. Zafrullah, hd1802, https://lohar.com/mithelpdesk/hd1802.pdf

[7] M. Zafrullah, On $*$-homogeneous ideals, https://arxiv.org/pdf/1907.04384.pdf