

QUESTION (HD 2006) You write in your paper, [Comm. Algebra 45 (2017), 5264-5282] the following:

"(To add to the confusion, Zafrullah [41] defined an integral domain to be a generalized Dedekind domain if every divisorial ideal is invertible. In [7], these rings were called pseudo-Dedekind domains in analogy with pseudo-principal ideal domains, i.e., integral domains in which every divisorial ideal is principal.)" So, are you a confirmed confuser? Is then your paper referred as [41] just bunk and is the paper referred as [7] the standard reference?

ANSWER: My name is Muhammad Zafrullah, so when my coauthors are "American Native Speakers of English", I don't get to write the paper, unless the "lead author is sick" and the other author (s) cannot manage. In this case both my coauthors were in good health, and I wish them good health. So, I didn't get to write the above comment. In fact I found out about it after the paper got published. (I usually started out a paper, giving clearly written scope of the new project and sent it to some "influential" folks to see if we could work together. And usually I got to work on something new and had little or no chance to look into what was going on. In other words, I did not micro manage how my coauthors dealt with the material. It has hurt me a bit, but when you are on a roll, you don't care much.) Anyway, here's my response.

Let's put down the part of the text that contains the comment you have brought up, so that everyone knows what we are talking about.

"An integral domain is called a generalized Krull domain (cf. [24, p. 524]) if it is a locally finite intersection of essential rank-one valuation domains. This terminology goes back at least to Griffin [27], and such rings were considered by Ribenboim [36]. Popescu [35] introduced the notion of a generalized Dedekind domain via localizing systems. Nowadays, the following equivalent definition is usually given: an integral domain is a generalized Dedekind domain if it is a strongly discrete Prüfer domain (i.e., $P \neq P^2$ for every prime ideal P) and every (prime) ideal I has $\sqrt{I} = \sqrt{(a_1, \dots, a_n)}$ for some $a_1, \dots, a_n \in I$ (or equivalently, every principal ideal has only finitely many minimal prime ideals). (To add to the confusion, Zafrullah [41] defined an integral domain to be a generalized Dedekind domain if every divisorial ideal is invertible. In [7], these rings were called pseudo-Dedekind domains in analogy with pseudo-principal ideal domains, i.e., integral domains in which every divisorial ideal is principal.)"

To facilitate the reading of the above passage, here are the references mentioned above:

[24] Gilmer, R. (1972). Multiplicative Ideal Theory. New York: Marcel Dekker.

[27] Griffin, M. (1968). Rings of Krull type. J. Reine Angew. Math. 229:1–27.

[36] Ribenboim, P. (1956). Anneaux normaux réels à caractère fini. Summa Brasil. Math. 3:213–253.

[35] Popescu, N. (1984). On a class of Prüfer domains. Rev. Roumaine Math. Pure Appl. 29:777–786.

[41] Zafrullah, M. (1986). On generalized Dedekind domains. Mathematika 33:285–295.

[7] Anderson, D. D., Kang, B. G. (1989). Pseudo-Dedekind domains and divisorial ideals in $R[X]_T$. *J. Algebra* 122:323–336.

[12] Zafrullah, On a property of pre-Schrier domains, *Comm. Alg.* 15 (1987) 1895-1920.

Now I named the domains generalized Dedekind (G-Dedekind) domains, because they appeared to be the closest to what Dedekind domains are and I do not know if my decision would change if I knew about Popescu's paper [35]. Finally [7] was "submitted" a year after the authors of [7], had seen my paper [41]. Now let me tell you the whole story.

I had left my job in Libya with the plan that I'd spend a year or two in London doing some research and looking for jobs and for a country that I could call my own. Joe Mott invited me to give some talks at FSU Tallahassee Fla. and in an effort to give me sufficient exposure Joe arranged for me to give talks at various universities, including University of Iowa, Iowa City, UNC Charlotte, NC and UT Knoxville, TN. (Actually folks who had seen my work were keen to see me in person. So everyone who Joe called agreed to have me visit them.) I gave a talk on [41] at Knoxville and there, on hearing the description of my results from David Anderson, over the phone, Dan told David that his student, B.G. Kang had also characterized G-Dedekind domains. I did not know what to say except that gee my hands are tied, the paper is in press and here are the galley prints. (Actually the paper was out at that time, only I had not gotten the reprints yet.) Now they submitted their paper late in 1987, rewriting my results in an expanded fashion and insinuating that "Zafrullah [12] has defined an integral domain R to be a $*$ -domain if $((\cap(a_i))(\cap(b_j))) = \cap_{ij}(a_i b_j)$ for all $a_1, \dots, a_n, b_1, \dots, b_m \in R$ (or equivalently, $\in K$). We observed that R is a $*$ -domain if and only if $(AB)^{-1} = A^{-1}B^{-1}$ for all finitely generated nonzero fractional ideals A and B of R . This lead (correction, "led" is the past of lead) us to investigate domains satisfying $(AB)^{-1} = A^{-1}B^{-1}$ for all nonzero fractional ideals. After our research was completed, we learned that Zafrullah [13] had independently obtained the equivalence of (1), (2), (4), and (5) of Theorem 2.8 and the result (proved in the final section) that R pseudo-Dedekind implies that $R[X]$ is pseudo-Dedekind." Hogwash, research was completed and they took almost a year fine-tuning it. Besides, going from a theorem in [13] to another, is the flimsiest of "explanations". I tell you from experience that if my paper had not appeared and I had agreed to let them join in as coauthors, my name won't be on the paper as a coauthor. At the most there would be a comment: The authors thank M. Zafrullah for piquing our interest in —, and for several helpful suggestions. (If you don't believe me, look at how they treat the $*$ -property. If you read [12], it was more extracted as a property of pre-Schreier domains than a mere definition.)

No sir, they published it because they could. Otherwise they were redoing, albeit in a more well-thought out fashion than I could, considering my circumstances. (Imagine if the roles were reversed, would "Muhammad Zafrullah" be able to publish in *Journal of Algebra*? The name Muhammad Zafrullah would be in the way to start with. I am thankful to my coauthors, especially Dan

and David Anderson. Whenever a paper came back in my face from a high tare journal, I would go and beg them to join in and they'd graciously help out. The resulting paper would appear in a similar or higher tare journal. Note that most of my joint papers with them are well-cited.)

Coming back to your question. No sir, I am not a confuser. I defined the domains to the best of my knowledge of English and of the domains. The domains satisfying $(AB)^{-1} = A^{-1}B^{-1}$ for all nonzero (fractional) ideals A, B are the closest to Dedekind domains. There is nothing pseudo about them. Popescu's "generalized Dedekind domains" can well be pseudo Dedekind, being a far removed relative of them, missing out on being completely integrally closed. I think Dan wrote that passage to cover his smelly behind. For, some unkind folks might take [7] to be plagiarism. (I think [7] puts [41] in a better light if you read [41] after [7], that is if the name Muhammad Zafrullah does not put you off.) If there is any justice in the world, [41] is where it started. Use whatever appeals to you more as a standard reference and let the better and more original work survive.

It may be that Dan wanted, also, to deepen the negative impact his pranks have had on my work for some other gains. (The dagger's in, let's give it a twist.) I had written a paper "On t -invertibility" with Saroj Malik and Joe Mott [MMZ, Comm. Algebra 16(1988) 149-170]. In it we had laboriously proved that D is a Krull domain if and only if each minimal prime over a proper nonzero ideal of type $(a) : (b)$ is t -invertible. This paper was submitted in July 1986 and we got acceptance in February 1987. Submitted in August 1987 by B.G. Kang was a paper that pre-empted [MMZ] and stole results from [41] stating them for pseudo Dedekind domains while they were proved for G-Dedekind domains in [41]. (I am not far wrong when I say that higher tare journals would rather publish plagiarized versions of Muhammad Zafrullah's work than publish Muhammad Zafrullah's work.) This paper appeared as [K, J. Algebra 124 (1989) 284-299]. In this paper Kang used the $*$ -property as if it was something he'd picked up by the roadside, or from his father's seachest. (Now read Theorem 1.10 of [41] and see how changing "G-Dedekind" to "pseudo Dedekind", results have been plagiarized in Theorem 5.1 of that paper.)

Now, Kang would not dare and he wouldn't have a clue if he had no insider help or support. Let me put it this way: Once I moved to the US and started working with Kang's benefactors, the heavenly revelations to the great South Korean Multiplicative Ideal Theorist died out and he resorted to writing insignificant papers such as [Comment. Math. Univ. St. Paul. 48 (1999) 19-24] using the definition of a t -class group as if he'd inherited it from some Japanese soldier. Now, Dan is already an editor of a South Korean journal and perhaps Dan is looking for a prize of some kind. Good luck to Dan, but whatever he gets won't be for Mathematics, it would be for treachery and for grave injustice done to me. The result of his tricks is that today South Koreans, and some Italians, are treating my work as their own property as if I were a Japanese soldier or some SS fellow from Germany.

(Having said and done all that, some of which I wouldn't ordinarily do, let me point out that there are a lot of examples of the same concept going under

different names. In some cases concepts are rediscovered and given different names and people deal with the situation by some times attaching with the terminology the name of the person who used it/defined it or by using their preferred terminology and mentioning that it is also called by so and so as such and such in [SOSO]. Here're a couple of fresh example.

Kesavan Thanagopal in his, Oxford D.Phil thesis entitled, "On the decidability of finite extensions of decidable fields" writes in a footnote: The notion of "generalized Dedekind ring" as we are describing in this paper appears to be specific to Ershov's works and should not be confused with the same notion as introduced by Nicolae Popescu in his work On a Class of Prüfer Domains (cf. [Pop84]) (i.e. [35] in which a domain R is defined to be a generalized Dedekind domain if and only if it is a Prüfer domain, $p \neq p^2$ for every non-zero prime ideal p of R , and every prime ideal of R is the radical of a finitely generated ideal. Yet another notion of "generalized Dedekind domain" has also been introduced by Muhammad Zafrullah in the paper On Generalized Dedekind Domains (cf. [Zaf86]) (i.e. [41]) in which an integral domain R is defined to be a generalized Dedekind domain if and only if for all non-zero fractional ideals I and J of R the following condition is satisfied: $(IJ)^{-1} = I^{-1}J^{-1}$. "A similar example comes from a highly influential paper [J. Alg ebra 320, 2907–2916 (2008)] by Evrim Akalan, who prefers the term G-Dedekind. She writes: Anderson and Kang [3] (i.e. [7]), and Zafrullah [19] (i.e. [41]) studied the question: if R is a commutative integral domain when is $(AB)^{-1} = A^{-1}B^{-1}$ for all non-zero fractional ideals A and B of R ? (1)

Authors call commutative integral domains satisfying (1), pseudo-Dedekind domains and G-Dedekind

domains, respectively. In this paper we study the analogue of this question for non-commutative maximal orders. So, Dan's hands went up in the air for nothing. There was no confusion.)

I hope I have answered your question fully and adequately.